NOTES ON REALIGNMENT OF CURVES FOR RAILWAYS



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Realignment of Curves

5.1 Maintenance of Curved alignment: The curved alignment differs from straight track in one important aspect i.e. the presence of lateral forces as centrifugal force, or centripetal force, depending on the speed of movement of train on the curve. The lateral forces cause wear and tear of the rail, sleepers and fastenings and also cause the track geometry to deteriorate. Therefore the curved alignment requires more maintenance compared to the straight track.

The aim of permanent way engineers while maintaining curved track is to ensure proper versines, gauge and cross-level. Abrupt changes in curvature and super-elevation on curves will result in poor riding comfort on the curves, increased wear and tear of rails, fittings and wheels, and may even be unsafe for the vehicles. The curves shall be maintained in such a manner that the versines and superelevation are varying gently and the super-elevation at each point should be appropriate for the curvature at that point.

5.2 Rectification of parameters in curve: As discussed above, the most important parameters of track in the curve to be monitored are: Gauge, Cross level and Versines.

- 5.2.1 **Gauge:** The procedure for rectification of gauge in curved track is similar to the procedure in case of straight track i.e. by making adjustments in the liners of the concrete sleepers. However, the gauge correction in the curves becomes slightly more difficult due to the wear in the gauge face of the outer rails. Beyond a certain limit, it will not be possible to rectify the gauge, and rails must be replaced or interchanged in this condition.
- 5.2.2 **Superelevation:** When the superelevation in curved track gets disturbed, the lower rail is taken as reference and the outer rail is raised. The lowering of any rail for the rectification of the superelevation defects is normally not done. However, while rectifying the cross levels, the vertical profile of the track is also to be kept in mind. The inner rail is taken as reference for the vertical profile of track.
- 5.2.3 **Versines:** As we know that versine is the parameter used for the measurement and rectification of the curve geometry. The versines are measured on shifting chord. This means that each point is involved in versine reading of three stations and therefore, the shifting of any point on the curve will affect the versine readings at three stations. Therefore, while rectifying the defects in the versine readings, we have to keep in mind the effect of changing the alignment at one point on the adjoining two points. Failure to do so will result in shifting of the problem to another point and the desired objective of alignment rectification will not be fulfilled. Due to this reason, rectification of versines is the most difficult part of curve maintenance.

The rectification of alignment of curve is called realignment of curves. The various cases necessitating realignment in field are:

- Local adjustment of curve: Quite often, in field, the curve gets disturbed in a small stretches due to presence of special features in track such as the level crossings, points and crossing, bridge, SEJ etc. In such a situation, it is not required to attend to the entire curve and the rectification can be done in affected length(s) only. This rectification of alignment locally is called **local adjustment of curve**.
- Attention to transitions: The versine and superelevation values change continuously in transition portions. Due to constant change of curvature, lateral forces in the transition portion are also variable. Therefore, even slight disturbance of geometry in transition portions will lead to fast deterioration of geometry. The transition portions of the curves are, therefore, required to be attended more frequently. This rectification of geometry in the transition portion of the curves alone is called attention to transitions.
- **Complete realignment of curve:** When the curve is disturbed in a large length and rectification of versine readings can be done only after considering the curve as a whole. The rectification of alignment of curve as a whole is called **complete realignment of curve**.

Increase in Transition length: The speed potential of the curve depends on the length of transition. When the speed potential of a line is to be increased or if there is a speed restriction on account of the inadequate transition length, we may plan the transition lengths of the curves to be increased. In such circumstances, change in length of transition will affect the versines of the circular portion of the curve also. Increased length of transition will also require increased shift, which varies with square of the length of transition. If we wish to accomplish the task with least amount of efforts, this problem of increasing the length of transitions becomes a special case of realignment of curves.

Note: 1. Sometimes, from speed considerations, we need to reduce the degree of the curve. Such a case requires the entire curve to be set again and is not in the scope of realignment.

2. The realignment is almost entirely a problem in open line, to be executed in block periods. However, the construction work such as gauge conversion, upgradation of lines etc may also require realignment. During construction work, the aim shall be to have as perfect geometry as possible. Therefore, normal constraints of open line such as block availability, resources etc shall not dictate the solutions in case the construction work is being done.

5.2.4 **Objectives of curve realignment:** Between any set of tangent tracks, infinite number of curves are possible, as we can see in figure 5.1. As we increase the degree of curve, the curve keeps on getting shifted towards the apex and vise versa. Therefore, we have large number of solutions to any realignment problem from which we have to choose the most

suitable solution. When carrying out realignment, we aim choose the curve amongst the various options which requires the least amount of efforts while meeting the desired objectives.

The curve realignment is done keeping the following in mind:

- The realignment of curve aims at improving the geometry so that the variations in versines is gradual and the maximum value of versine is within limits prescribed for the curve.
- The original designed versines in the curve are a good guidance when deciding the correction in versines, but the same cannot be the main



criteria and other aspects have to be seen in deciding the proposed versines. In some cases, trying to restore the original geometry may be very tedious and costly, not worth the efforts, or it may not be feasible due to subsequent developments such as insertion of points and crossings or OHE masts, signals or other installations. Therefore, the aim of realignment of curves is not to restore the curve to original geometry, but to desirable geometry.

5.3 Defects in Versines: If the versines in a curve are bad, the same can be identified by-

- a) Unsatisfactory running on the curves by engine foot plate/ last vehicle inspection.
- b) Unsatisfactory readings as per TRC/OMS/ Oscillograph runs
- c) Visible observation during routine push trolley/foot inspection.
- d) Measurement of versines of a curve.

In a curve, the originally designed values of the versines for a curve are not as important as the station to station-to-station versine difference.

Speed Range	Limits of station to station variation (mm)
120 kmph and above	10mm or 25% of the average versine on circular curve whichever is more
Below 120 kmph and upto 80 kmph	15mm or 25% of the average versine on circular curve whichever is more
Below 80 kmph and upto 50 kmph	40mm or 25% of the average versine on circular curve whichever is more

The service limits for the station to station versine variations laid down in IRPWM are:¹

In case the exceedence of the above limit is observed during an inspection, local adjustments may be resorted to in case where the variation of versines between adjacent stations is only at few isolated locations, at the earliest possible. If more than 20% of the stations are having versine variation above the limits prescribed, complete realignment of the curve should be planned within a month.

Note for Maintenance: The limits of versine variation given above are for safety and this is demonstrated by the fact that one month is given even after the values are exceeded at more than 20 % of the stations.

5.4 Record of Curve Survey: The curves shall be measured as per schedule laid down in IRPWM (Para 107(4), 124(4), 124 A(4), 139(3) and 139 A(4), chapter I) or whenever the curve appears to be unsatisfactory during inspections. The record of curve survey for realignment shall be kept in a curve register. The record should be collected in the following form:²

Curve from km.....to km.....to km..... Between station......and station..... Date of survey...... Jurisdiction of Assistant Engineer/Permanent Way Inspector

JUNSCICTION OF ASSISTANT EN	igineer/Permane	ant way inspector.	
Station at Half-chord intervals	Versine in (mm)	Cant existing	Remarks regarding restrictions to slewing
-2	-3	Zero	Points and Crossing at station no -2
-1	+2	-2 mm	
0	0	Zero	
1	2	5mm	
2	4	10mm	
3	4	20mm	
4	10	25mm	
5	11	29 mm	
6	23	35 mm	
7	36	40 mm	
8	28	40 mm	

¹ IRPWM para 421

² Para 422, IRPWM

9	35	38 mm	
:	:	:	
20	23	41 mm	Girder Bridge, obligatory point. Maximum slew ±50 mm
21	31	43 mm	
22	33	45 mm	
:	:	:	
40	38	38 mm	High Bank, 4 m height, between station nos 40 to 60
:		:	

The above record shall mention all important factors which might be relevant to the realignment such as location of track features such as points and crossing, girder bridge, level crossing etc, high bank, deep cutting, structures nearby such as platforms, road over bridge, building, etc. If, in some case, it is felt that tangent track in the vicinity of the curve is not having proper alignment, and the curve has got extended into the tangent track, a few stations beyond the curve shall be included in versine readings taken for the curve so that these are rectified along with the realignment of curve. In any case, whenever we are measuring the curve, the same shall be measured upto 2 or 3 stations ahead of and 2 or 3 stations beyond the curve so as to ascertain that the tangent track is actually correct and the curve has not extended into the tangent track.

5.5 Rectification of Curved Alignment: When rectification of alignment is required to be done in the straight alignment, the same can be done using theodolite or fishing cord. We can take any two points on good alignment and proceed to set out the alignment of all point sin between. PWIs are even able to carry out a reasonable job of rectification of the alignment in straight by eye-sight only.

However, when the rectification of the alignment in case of curves comes, the mind is not able to comprehend the alignment by eyesight and **no attempt shall be made to rectify the track alignment in curve by eye-sight**. The rectification shall be done only by proper measurements using theodolite, fishing cord etc. Further, as discussed above, the effect of the rectification of track alignment at a point on the adjoining stations must be considered and hence **proper computations must be done before taking up the rectification of curved alignment**. Before we proceed further, a few terms associated with the realignment need to be defined:

- 5.5.1 **Slewing:** Shifting the alignment of track is also called slewing. The slewing can be in either direction, outside of the curve or inside of the curve. When the track is slewed outwards, away from the center of curve, it leads to sharpening of the curve as the radius reduces and curvature increases. On the other hand, the shifting of the track inwards, towards the center of curve, results in easing of the curve as the radius increases and curvature reduces.
- 5.5.2 **Sign convention for slews:** In this book, as a standard sign convention, the <u>outward</u> <u>slew is considered as positive slew</u> whereas <u>inward slew is considered as negative slew</u>.
- 5.5.3 **String lining operation:** The method of determining the corrections to be made in the alignment of a curve, after measuring the versines and the rectification of the alignment thereafter is called string lining operation.

Note: The procedure given in para 5.6 and 5.7 below is the basic mathematical treatment to the subject of realignment. The computer programs use these for computations. Knowledge of the same will enable the engineer to be able to better understand and control the output of computer programs. Since these days computers are widely available and it is not expected that the curve realignment problem will be solved by hand even in field, it is requested that the field engineers

go through the para 5.6 for their information and seek to understand the basics rather than learn the procedure by heart.

- **5.6** String Lining for Rectification of curved alignment: The string lining method has the following stages:
 - Measurement of versines of existing curve.
 - Consideration of Obligatory points.
 - Determination of the revised alignment and computation of slews.
 - Slewing of the curve in field to the revised alignment.
- 5.6.1 **Measuring of existing versines:** Before starting the measurement of versines, the gauge correction shall be carried out. After gauging is done, the versines are measured. As the geometry tends to get disturbed in the tangent track adjoining the curve, a few stations ahead of curve and behind the curve on the tangent track shall also be included in the versine readings. These readings will also help when the length of curve may have to be increased on either side for proper rectification of the curve geometry.
- 5.6.2 **Consideration of Obligatory Points:** While measuring the versines, the features that restrict the extent to which the track can be slewed in either direction shall also be noted down. Such points are called obligatory points. The locations where such restrictions are there include girder bridges, point and crossings, adjoining track, permanent structure adjoining the track or limitation of railway land boundary, OHE mast, signal post, platform, FOB/ROB column/abutment etc. The obligatory points may not permit any slew or only restricted slew. Many obligatory points have restriction in slew only on one side. The maximum amount of slew is normally restricted in the OHE area to ±50mm without alteration in OHE.

It must, however, be remembered here that any obligatory point is obligatory only in respect of costs and benefits. This means that there is a cost involved in removing any obligatory point. The benefit of improvement in geometry, and thereby the speed/comfort of movement, is to be weighed against the cost required to remove the restriction before taking any decision regarding any track feature before declaring the same as obligatory point. If the shifting of curve gives benefit such as removal of a permanent speed restriction etc, it will be worthwhile to put in efforts and shift the feature which is restricting the shifting of the curve.

- 5.6.3 **Working out the revised alignment and computation of slews:** Based on the measurement of existing versines, the proposed versines have to be chosen. To choose the revised alignment, the exercise of computing the first and second summations of the versine readings is to be done.
- 5.6.3.1 **First Summation of Versines:** The first summation of versines at a station is the sum of all versines upto the station. In these notes, the same is represented as FSV with subscript 'e' for existing and 'p' for proposed versines.

Station No	Versine	FSV
0	V ₀	-
1	V ₁	V ₀
2	V ₂	V ₀ + V ₁
3	V ₃	$V_0 + V_1 + V_2$
		•
•		

Table 5.1: Computation of first summations

•	•	
n	Vn	V_{0+} V_{1+} V_{2} + $V_{n-2}+$ V_{n-1}

The first summation computation procedure is shown in example 5.1 below.

5.6.3.2 Physical Meaning of First Summation of Versines:

Figure 5.2

As seen in table 5.1, FSV upto a station is the sum of all versines upto that station i.e. for station 'n', $FSV=V_0 + V_{1+}V_2 + ... + V_{n-2} + V_{n-1}$.

If we look at the figure 5.2. it is clear that if we divide the versine diagram into histograms with each segment equal to the distance between the stations, the area of the versine diagram is also $V_0 + V_1 + V_2 + \dots + V_{n-2} + V_{n-1}$. Here the distance between the stations has not been considered separately i.e. First summation of versines upto a station represents the area of versine diagram



If station to station distance is taken as unit,

Area of each histogram segment= Ordinate at center i.e., V_0 , V_1 , V_2 , ..., V_{n-1} etc Total Area of the versine diagram= Sum of areas of each histogram segment

$$V_0 + V_1 + V_2 + \dots + V_{n-1}$$

Lever Arm for each histogram segment= n-station number \therefore Moment of versine diagram about station n= n* V0+(n-1)* V1+ (n-2)V2++ 2* Vn-2+ Vn-1

upto the station(in station distance units).

5.6.3.3 **Second Summation of Versines:** The second summation of versines at a station is the sum of all first summation of versines upto the station. In these notes, it is represented as SSV with subscript 'e' for existing and 'p' for proposed versines.

Station No	Versine	FSV	SSV
0	V ₀	V ₀	-
1	V ₁	$V_0 + V_1$	V ₀
2	V ₂	$V_0 + V_1 + V_2$	$2^* V_0 + V_1$
3	V ₃		3* V ₀ +2* V ₁ + V ₂
		•	•
n	V _n	V_{0+} V_{1+} V_{2} + $V_{n-2}+$ V_{n-1}	$\begin{array}{c} n \ ^*V_0 + (n\text{-}1) \ ^*V_1 + \ (n\text{-}2) \ ^*V_2 + \ \ldots \\ + \ 2 \ ^*V_{n\text{-}2} + \ V_{n\text{-}1} \end{array}$

The second summation computation procedure is shown in example 5.1 below.

5.6.3.4 Physical Meaning of Second Summation of Versines: As seen in table 5.2, FSV upto a station is the sum of all versines upto that station i.e. for station 'n', SSV= n* $V_0+(n-1)^* V_1+ (n-2)V_2+ \dots + 2^* V_{n-2}+ V_{n-1}$. If we see the figure 5.2 again, the lever arm for V_0 upto the last station is n. The lever arm for V1 is (n-1) and so on and the

lever arm for Vn-1 is 1. This means that the expression for second summation is also the moment of the histogram derived from the versine diagram about the last station. (Again the distance between the stations has not been considered separately) i.e. SSV upto a station represents the moment of versine diagram upto the station (in station distance units).

- 5.6.3.5 **Properties of Versines of a curve:** The proposed versines are to be chosen keeping the following basic properties of versines of curved track in mind:
- a) First Property of Versines of any Curve: Slewing a station on curve affects the versines at the adjoining two stations by half the amount in opposite direction. Let us examine what happens when we slew a point on the curve. Let us consider a curve with stations A, B, C, D and E at any point on the curve each at one station unit distance between them. If we slew the station C to C' without disturbing any other point on the curve, the versines are affected at station B, C and D, as shown in fig 5.2.

Let the point C be slewed outwards by an amount cc'. The chord AC gets shifted to AC'. The change in the versine at B is bb'.

Consider \triangle Acc', since the radius of railway curves is very large, we can consider cc' as parallel to bb' (the figure 5.3 is not drawn to scale and the lines do not appear to be parallel but in field, the assumption is quite valid).



∴ The triangles Acc' and Abb' are similar, i.e. $\frac{bb'}{cc'} = \frac{AB}{AC}$.

Now, since the radius of the railway curves is quite large as compared to the distance between the stations, the chord Ab \sim AB and if c is the chord length, AB =c/2 and AC=c

 $\therefore \frac{bb'}{cc'} = \frac{1}{2}$ i.e. the slew at station B is half that at station C. Also, looking at fig 5.2, it is clear that

when the station C is slewed outwards (versine at station C increases), chord AC shifts towards the station B and vice versa.

Similarly, the change in versine at station E also changes by an amount equal to half the amount at D, and the change will be in opposite direction to the change at station C.

Therefore it is clear that whenever we slew the curve at a station, the versines at the stations at either side get affected by half the value of slew in opposite direction. Whenever we disturb the curve, we have to keep this property in mind. For example, if we slew curve at station n by 10mm outwards i.e. versine increases by 10mm at station n, the versines at station (n-1) and (n+1) reduces by 5mm each.

b) <u>Second Property of Versines of any Curve:</u> For equal chord lengths, sum of existing versines shall be equal to sum of versines. The same can be proved as follows: Let us consider a curve with only 3 stations as shown in fig 5.4.

If chord length is c, the versines can be worked out as -





$$\begin{split} V_o &= c/2 \tan \propto \cong c/2 \propto \\ V_1 &= c/2 \tan \beta \cong c/2 \beta \\ V_2 &= c/2 \tan \Upsilon \cong c/2 \Upsilon \\ \therefore \Sigma V &= V_o + V_1 + V_2 \\ &= (c/2)^* \propto + (c/2)^* \beta + (c/2)^* \\ &= (c/2)^* (\propto + \beta + \Upsilon) \end{split}$$

Now, in a triangle, it can be demonstrated that the external angle is equal to the sum of the opposite two angles. \therefore If we consider Δ IHJ, < RIJ = < IHJ + < IJH = $\propto + \propto = 2 \propto$

Similarly, in Δ JKL, < RKJ = 2 Υ and in Δ JRK, Δ =< RJK+<RKJ

Υ

i.e.
$$\Delta = (2 \propto +\beta) + (\beta + 2\Upsilon)$$

»
$$\Delta = 2[\infty + \beta + \Upsilon]$$

»
$$\Delta/2 = [\infty + \beta + \Upsilon]$$

Substituting in the equation above,

$$\therefore \Sigma \mathsf{V} = (\mathsf{C}/2)^* (\Delta/2) = \frac{c \times \Delta}{4}$$

This means that the sum of versines for a curve depends only on the chord length and the deflection angle (i.e. a curve between the same set of tangents), This proves the first principle of the realignment of curves for a curve with three stations. QED.

From the first property of the versines of any curve, it comes out that the sum of the versines in any curve will remain constant even when it is disturbed at any one point. Starting from a three station curve shown in fig 5.4, we can progressively slew the curve, one station at a time and the sum of versines will always remain constant, regardless of the actual length/ no of stations. This property means that whatever be the actual shape of the curve and/or the location of the starting/ end point of the curve, the summation of the versines remains constant.

As a corollary to the above property, whenever we choose any solution to realignment problem and propose new versines, the **sum of the proposed versines has to be kept same as that of existing versines.** This property is used when we choose a solution to the realignment problem.

c) <u>Third property of Versines of any Curve</u>: Twice the second summation of the difference of proposed and existing versines upto a point gives the slew at a station: To demonstrate this property, let us draw the curve from station no -1 to station no 3 and only one tangent in fig 5.5.



The fig 5.5 is showing deflection of the curve from the straight (tangent). The curve is having stations marked -1, 0, 1, 2 and 3. AB is the tangent track and BK is the extension of the same. Points E, I and N correspond to stations 1, 2 and 3 on the curve. AE is the chord stretched

to measure the versine at station 0. Similarly, BI and EN are chords stretched to measure versines at station no 2 and 3 respectively.. Line, BEHL, joining station 0 and 1 and extended further is drawn. Another line EIM, joining station 1 and 2 and extended further is drawn.

• Let the versine at station 0, i.e. BC be V_0 .

By geometry, consider similar triangles ABC and ADE.

Since AD= 2 station units and AB=1 station unit, AD= 2xAB

 \therefore DE = 2xV₀.

Further consider similar triangles BEF and BGH.

Since BD= 1 station unit, and BG=2 station units, BG=2xBD,

 \therefore GH=2xDE. i.e. GH=2x2xV₀ and

In similar triangles BEF and BKL,

since BD= 1 station unit, and KL=3 station units, BK=3xBD,

∴ KL=3xDE. i.e. KL=3x2xV₀.

 \circ In the similar manner, let the versine at station 1, i.e. EF be V₁.

By geometry, consider similar triangles BEF and BHI.

Since BH= 2 station units and BE=1 station unit, BH= 2xBE

 \therefore HI = 2xV₁.

Further consider similar triangles EHI and ELM.

Since EH= 1 station unit, and EL=2 station units, EL=2xEH,

 \therefore LM=2xHI. i.e. LM=2x2xV₁.

 \circ And, let the versine at station 2, i.e. IJ be V₂.

By geometry, consider similar triangles EIJ and EMN.

Since EM= 2 station units and EI=1 station unit, EM= 2xEI.

 \therefore MN = 2xV₂.

• Now let us consider the deflection of the curve form the straight (tangent):

Offset at station 0: 0

Offset at station 1: $DE = 2xV_0$.

Offset at station 2: G-H-I ~ GH+HI= $2x2xV_0 + 2xV_1$. (There is a small error here but since the railway curves have very large radii, the error is small and can be neglected).

Offset at station 3: K-L-M-N ~ KL+LM+MN= $3x2xV_0 + 2x2xV_1 + 2xV_2$.

Continuing the trend further,

Offset at station n: $nx2xV_0 + (n-1)x2xV_1 + (n-2)x2xV_2 + ... + 2x2xV_{n-2} + 2xV_{n-1}$

 \Rightarrow Offset at station n: 2x[nV₀ + (n-1)V₁+(n-2)xV₂ +... + 2xV_{n-2} +V_{n-1}]

Now, since the equation in the bracket, i.e. $nV_{o+}(n-1)V_{1+}(n-2)xV_{2+}\dots+V_{n-2}+V_{n-1}$ is the second summation of the versines of a curve.

 \Rightarrow Offset at station n: 2 x [Second summation of versines upto station n]

This means that twice the second summation of versines of a curve up to a station represents the offset of curve from tangent at that station.

If we look at the fig 5.6, the solid curve represents curve as measured in field. Let the dotted line



represent the desired geometry of the curve. Now the offset of the existing (solid) curve form straight is given by twice the second summation of the existing versines. Whereas the offset of the proposed (dotted) curve from straight is given by twice the second summation of the proposed versines. This means that if we calculate twice the difference of second summations of the existing and proposed versines, we get the slew at a station. This

is the third property of the realignment of versines of any curve.

d) Fourth property of Versines of any Curve: The second summation of the difference of proposed and existing versines at first and last stations shall be zero: This property is not sacrosanct but is meant to isolate the curved track from straight alignment. This ensures that while carrying out the realignment, we have isolated the curved track and are trying to solve the alignment problem of this portion of the track alone. If the slew is not zero at the first and last stations, it would imply that we are attempting to realign the straight portion also. In such a case, the problem in the curved track will be transferred to the straight portion of the track, and as we understand the problem will not be possible to be solved unless the complete straight is aligned with this changed geometry.

Here the first and last stations do not mean the start and end of the curve but the first and last station included in the curve survey. It is for this reason that it is mentioned in para 5.5.1.2 that the disturbed components of tangent track (straight) adjoining the curve which are lying outside the original curve length shall also be included in the curve survey.

If it is found that the tangent track (straight) is not in correct alignment, the same may be rectified using theodolite or fishing chord or by sight and the problem of realignment of curve shall be isolated. And therefore, we have to ensure during solving the realignment problem that **the second summation of the versines at the first and the last stations shall be zero.**

- 5.6.3.6 **Proposed Versines for the String Lining Operation:** Based on the above properties of versines of any curve, the proposed versines shall be designed. The curve measured in field is termed as disturbed curve and the desired versines are called proposed versines. The steps in choosing proposed versines are as follows:
- a) The versines over the disturbed curve are tabulated and summed up.
- b) As per the second property, the proposed versines shall be chosen close to the average of versines over the circular portion of the disturbed curve. The versines on transitions are then designed with a uniform rate of variation over the stations on the proposed transitions commencing at or near the apparent tangent points at both ends. The versine distribution shall be as shown in figure 5.7.



Note: The shape of the proposed versine diagram as above shall be as close as possible to the theoretical trapezoidal distribution.

- c) If there is any small difference between the sums of the existing and proposed versines, due to rounding off errors, the balance of versines after deducting the total of proposed versines from the total of existing versines shall be distributed symmetrically over the centerline of the curve.
- d) Difference between the existing and proposed versines shall be found for all the stations.
- e) The first summation of the difference of the proposed versines and existing versines shall be found out. If the calculations are done correctly, the first summation shall be zero at the last station, in accordance with the first principle of curve realignment.
- f) Then the second summation of the difference of the proposed versines and existing versines shall then be found out.

If the proposed versines are correct, the second summation at last station shall be zero in accordance with the fourth property of the versines of a curve. However, since the curve as measured is disturbed and the solution that we have assumed is ideal trapezoidal distribution of versines, there are chances that the solution assumed is not the correct one. Quite often, a residual slew either outward or inwards at the last station of the curve is left. This residual versines is required to be eliminated by using what is known as "**correcting couple**". A correcting couple is a set of two equal and opposite corrections to versines applied at two stations such that the residual of second summation at the last station is reduced to zero.

What is physical meaning of correcting couple?

• When there is residual second summation of versine difference at the last station, it means that that the second summations of the existing and proposed versines are not equal, I.e. the center of gravity of the exiting versine diagram does not coincide with the proposed versines. The correcting couple (which is a small correction in the proposed versine diagram (fig 5.8)) means a small transfer of the area of versine diagram such that the centers of gravity of the existing and altered proposed versine diagrams match.



• Another way of looking at the things is if we look at the curve in plan as shown in fig 5.9. When the proposed versines is not the correct solution to the realignment problem, the second summation of the proposed curve at the last station (proposed slew from tangent) is not coinciding with the second summation of the existing curve(existing slew from tangent). The correcting couple changes the second summation at all the stations subsequent to the first station of application, i.e. the offset of the curve changes at all these points. Therefore, the correcting couple means that the proposed curve is held at a point and turned about this point in such a manner that the degree of the curve remains the same at the other points but the end of the proposed curve meets the existing curve.

A few questions regarding how to choose correcting couples must have arisen in the minds of the readers, which are answered below:

Why correcting couple?: The correction in proposed versines can be accomplished by changing the proposed versines and doing the complete calculations again. However, we are not sure as to what might be the correct solution for the proposed versines, we might have to do a number of iterations. For ease of calculations, we separate out the calculations for the corrections in the form of correcting couple.

Why correcting couple has to be equal and opposite?: The sum of the versines must remain constant as per the first property of versines of a curve, hence whatever correction is applied, must have sum zero. Therefore, the correcting couple has to be equal and with opposite signs.

What shall be sign of the correction applied first?: In the tabulated versines, the two elements of correcting couple are applied one above the other. As discussed above, the two have to be of opposite signs. The correction applied above the other must have the sign opposite to the residual second summation value at the last station so that the second summation of the correcting couple is having sing opposite to the residual second summation of proposed versines.

Where shall correcting couple be applied?: The capacity of a correcting couple to reduce the residual slew at the last station depends on the distance between the two corrections. Therefore, the correction shall be applied as far away from each other as possible.

The correcting couple is to be chosen in such a manner that the residual second summation of the versine difference and the correcting couple at the last station gets reduced to zero.

Principles of choosing correcting couples: To summarise, the correcting couple shall be chosen such that

• These are equal and opposite.

• If the residual second summation of versine difference at the last station is positive, a negative correction is applied at the initial stations of the curve and a positive correction is applied at the latter stations of the curve, such that the second summation of the correcting couple is negative. For a negative residual value of second summation of versine difference, the corrections with opposite signs shall be applied.

• The correcting couple disturbs the trapezoidal distribution of the proposed versines, and consequently the correction shall be as small as possible in value. For this purpose, the two corrections are applied at stations which are as far apart as possible. If the corrections are applied farther apart, the second summation of the correcting couple will be higher.

• If it is found that the one correcting couple is not sufficient to counteract the residual second summation of the versine difference at the last station, apply more correcting couples. The further couples are also to be applied in as small absolute value as possible and with as much distance between them as possible.

g) Above is continued till the second summation of the correcting couple becomes equal to the residual value of the second summation of versine difference at the last station. Now the second summation of the versine difference and second summation of the correcting couple are summed up. This gives the half slews at each of the stations in accordance with the third principle of realignment of curves. The slews at all stations are found by doubling the half slews so found out.

5.6.3.7 **Limitation of choosing proposed versines manually:**

- 5.6.3.7.1 The above procedure of choosing the proposed versines manually, without any mathematical assistance can be used conveniently only when the length of curve is small or if we have to tackle small portion of the curve. However when we take up longer curve lengths, choosing the proposed versines as above becomes difficult.
- 5.6.3.7.2 Further, in field, the curves often get too disturbed and shifting the beginning and end of curve may be required so as to have the proposed curve as close as possible to the shape of the existing curve, and we get minimum slews. This cannot be done if we use the method described above.
- 5.6.3.7.3 It requires tedious repeated iterations to design a correct proposed versine and to commence the curve at the appropriate point for the versine chosen such that:
 - a.) There is no residual slew at the other end, and
 - b.) The maximum slew, either outside or inside, does not exceed the practical limits, say 150mm.

To achieve these objectives, we must use a mathematical approach for designing the proposed versines. This is called optimization of the realignment solution.

5.6.3.8 Optimization of curve realignment solution:

- 5.6.3.8.1 If the realignment is attempted by the procedure outlined above, one of the following situations may arise:
 - **Improper choice of Beginning of Curve:** When the proposed versine chosen happens to be correct but the BC chosen is not correct, it results in heavy residual slew at the end, as shown in fig. 5.10.



Fig 5.10: Effect of choosing different beginning of curve

In the figure above,

Existing curve shown in solid line and proposed curves shown in dotted

- **BC: Beginning of Curve**
- EC: End of Curve

CC: Center of Curve

- 1 1 : Disturbed curve
- 2 2 : When BC chosen happens to lead the correct BC
- 3 3 : When BC chosen happens to trail the correct BC
- 4 4 : Proposed curve with correct BC and EC symmetrical about CC

Note: Curves 3-3 & 2-2 are not symmetrical about CC

• **Improper Choice of versine:** When the BC chosen is correct but the proposed versine chosen is not correct, it results in heavy slews outward or inward over the entire curve. The effect of different versine readings is shown in fig 5.11.



Fig. 5.11: Effect of choosing incorrect versine

- 11: Disturbed curve
- 22: Proposed curve when versine chosen is less than required
- 33: Proposed curve when versine chosen is more than required
- 44: Proposed curve with correct versine

Note: All the three curves, 2-2, 3-3, 4-4 are symmetrical about (CC). The curves 2 2 and 3 3 are away from the existing curve and hence the slews are more.

5.6.3.8.2 **Choosing Correct Beginning of Curve and Proposed Versines:** As discussed above, success in obtaining the most suitable revised and rectified alignment in string lining operations depends not only on the correct choice of versines but also on the correct choice of the starting point.

To enable decision making regarding the correct BC and versines, concept of fixing a point on the curve, usually center of curve (CC) is used. The following procedure is adopted for the same:

• Step I: Find the chainage of the centre of the existing curve (CC): The center of curve used for the realignment solution is not the physical center of the length of curve. The CC is the x-coordinate of the center of gravity of the versine diagram.

The center of gravity of existing versine diagram can be determined by dividing the moment of the versine diagram about the last station by the area of the versine diagram. In order to get the value of CC in reference to the first station, the center of gravity thus found has to be deducted from n(last station of the existing curve).

i.e.,
$$CC = n - \frac{SS \ Second \ Summation \ of \ V_e(SSV_e) \ up to \ last \ station(n)}{FS \ First \ Summation \ of \ V_e(FSV_e) \ up to \ last \ station(n)} \dots (5.1)$$

NOTE: For making the correct choice of BC and proposed versine, we assume that the CC is not disturbed. Therefore, the CC found out is step I is also the CC for the proposed versine diagram.

- Step II: Find the offset from tangent at CC for existing curve: As per the third principle of curve realignment, the offset of a station from the tangent is given by the second summation of the versines upto that station. Using this principle, the offset of the CC for existing curve can be found out from the second summation worked out for the existing versines. If CC happens to fall between two stations, the offset at CC shall be found by interpolation of the values at the adjoining two stations.
- Step III: Find the offset at CC for the proposed curve: As we desire to keep the center of curve undisturbed, the offset at CC for the proposed curve shall be equated with the offset at CC for the existing curve. (*Physically, this means that we aim at maintaining constant center of gravity of the curve. It may be recalled that this is what we aimed at when the correcting couple was being applied in para 5.6.3.6(f). Here the same is being done before the versines are chosen*). The offset at CC for the existing curve has already been found in step II and the offset for the new curve can be found out if we consider **equivalent circular curve**. Equivalent circular curve is the proposed circular curve if there were no transitions. The equivalent circular curve is shown as dotted line in fig 5.12. The equivalent circular curve is used a sit is easier to do calculations for the same.



Fig 5.12: Equivalent circular curve

Let the length of the proposed equivalent circular curve be N station units. Let the uniform versine in circular curve be V and L be the length of transition curve in station units

If T is the tangent length upto CC, for the equivalent circular curve, the offset at center of T^2

curve: Oc= $\frac{T^2}{2R}$

Since the actual proposed curve is also having transitions at either end, the equivalent curve shifts inwards when the transition is introduced by an amount equal to shift, S.

Using equation (1.21), Offset at CC for the proposed curve= Oc + S = $\frac{T^2}{2R} + \frac{L^2}{24R}$(5.2)

The minimum length of transition for any curve is known from the speed potential of the section, and the designed cant actual/ cant deficiency values.

The value of L chosen shall be

-more than the minimum required from the speed considerations

-based on the trend of the existing curve versine readings.

• Step IV: Equate the offset at CC for the existing and proposed curves:

From equation (1.2), versine V =
$$\frac{C^2}{8R}$$
,

Considering C = 2 station units, V= $\frac{2^2}{8R} = \frac{1}{2R}$(5.3)

For the railway curves, the radius of curve is very large, so the tangent length, T can be approximately considered to be equal to half the curve length without appreciable error

i.e. T~
$$\frac{N}{2}$$
.

:. The equation (5.2) reduces to Offset at CC = $\frac{VN^2}{4} + \frac{VL^2}{12}$ (5.4)

The versines are designed by slope method and it is assumed that the versine starts from zero to the slope of the versines in the transition portion. However, in actual practice, the vehicles have bogies and these will moderate the effect of any sudden introduction of the versines and the actual bogie traveling over the transition experiences slightly different versines. Therefore, the equation (5.4) gets slightly altered and the final value comes to

$$O_c + S = \frac{VN^2}{4} + \frac{V(L^2 - 4)}{12}$$

i.e. Offset at CC for proposed curve $=\frac{V^2 N^2}{4V} + \frac{V(L^2 - 4)}{12}$ (5.5)

The total versine for the equivalent circular curve is V * N, From the first principle of curve realignment, the sum of versines for any curve between the same set of tangents for equal chords is equal, therefore, sum of versines of the existing curve.

Therefore, V * N = FSVe....(5.6)

Substituting from eqn (5.6) in eqn (5.5), we get

Offset at CC for proposed curve = $\frac{(FSV_e)^2}{4V} + \frac{V(L^2 - 4)}{12}$

Since, offsets for the existing and proposed curve are to be equal, the LHS of the equation is known from step II above,

Offset at CC for existing curve = $\frac{(FSV_e)^2}{4V} + \frac{V(L^2 - 4)}{12}$ (5.7)

We have already found out the offset at CC for the existing curve in step II, para 5.5.5.2, there is only one variable i.e. V in the above equation. The quadratic equation (5.6) can be solved to get the proposed versine in the circular portion of the curve.

• Step V: Find out the correct length of curve: In step IV, the value of the proposed versine, V, in the circular curve is found out. Now, the length of equivalent circular curve can be found out using equation (5.6):

FSVe= N x V, i.e. N = FSVe/ V(5.8)

As shown in fig 5.11, the length of the proposed curve is found out by adding the designed transition (L) in station units to the circular curve length N'=N+L.

• Step VI: Find out beginning and end of curve: Now, Chainage of the beginning of the

proposed transitioned curve (BC) is then found out as (BC) = (CC) - $\frac{N+L}{2}$(5.9)

And, Chainage of the end of the proposed transitioned curve (EC) is found out as EC = $(CC) + \left(\frac{N+L}{2}\right)$(5.10)

 Step VII: Design of proposed versines: The proposed versine diagram shall be as close to the trapezoid as possible. The versines in the circular portion of the curve shall be V and for the transitions, the proposed versines are found out using the versine slope i.e. $\frac{V}{L}$. The

versines are proposed for each station by rounding off the values obtained by the versine slope. In Case BC and EC do not fall on any station, the versines at the first/last stations can be found out by interpolation. Versines at other stations on the transitions can be designed by successive addition of value of versine slope, till the value reaches just short of designed versine for circular curve. The final versine diagram of the proposed versine diagram is shown in fig 5.13.







Since the versines are to be proposed in whole numbers or at the most to first decimal point only, there will be some rounding off involved. There might be some small error in the sum of the versines so designed versus sum of versines for the existing curve. Adjustment of this minor difference may be done symmetrically at appropriate number of stations either near about the center of curve or at the junction of circular and transition curves, depending on the total error. This adjustment shall be as small as possible.

Now that we have designed the proposed versines, further computations are done as per the procedure given in para 5.6.3.

5.6.3.8.3 Passing the Curve through a desired point anywhere on the circular portion other than Center of Curve (CC):

The curve realignment problem cannot be solved without considering the site constraints. As discussed in para 5.5.1.1 above, it might not be possible to shift the curve at certain points called obligatory points. Let us take a curve, where X is the station on curve at which no slewing or shifting is possible.

As per the third principle of curve realignment, the offset of a station from the tangent is given by the second summation of the versines upto that station. Therefore, offset at station X for the existing curve can be found out from the second summation of the existing versines upto station X.

Using the same analogy as in 5.5.5.2 to 5.5.5.4 and eq (5.6), offset at station X for the proposed T_r^2 ($L^2 - 4$)

curve= $O_x + S = \frac{T_x^2}{2R} + \frac{(L^2 - 4)}{24R}$, where T_x is the length of the tangent upto X.

And since V= 1/2R, offset at station X for the proposed curve= $VT_x^2 + \frac{V(L^2 - 4)}{12}$

In this case, $T_x = N/2 + (X-CC) = FSVe/2V + (X -CC)$

: Second summation of the existing versines upto station X=

$$V\left(\frac{FSVe}{2V} + (X - CC)\right)^2 + \frac{V(L^2 - 4)}{12}....(5.7)$$

This gives us a quadratic equation for proposed versine for passing the curve through station X of the existing curve. After having designed the proposed versine, BC, EC and versines over transitions can be found out as explained earlier in para 5.5.2.2.4 to 5.5.2.2.7. The procedure for the same is explained in the example 5.4.

5.7 Change in Transition length: The need for change in transition length is discussed in para 5.2.3. Increased length of transition means increased shift, which varies with square of the length of transition. Such shifting of the entire curve will require considerable energy if the curve length is more. Sometimes, such solution may not be feasible in open line and it will be desirable to have a solution which has minimum slews.

The additional shift of the curve on account of the increased transition length takes place away from the apex of the curve. If the degree of the circular curve is increased, the curve will be closer to the apex, as shown in fig 5.1 and 5.11. Therefore, the slews for this case can be reduced by making the curve sharper and shifting the entire curve in the opposite direction. In this solution, the higher slews over the increased transitions can not be avoided but the additional shift due to increase in transition length over the circular portion is reduced.

In this case, the calculations shall be done as per the procedure outlined in the para 5.6. The increased transition length is considered in the procedure by choosing appropriate length of transition in station units. When we optimize the solution by keeping the slew at center of curve as zero, the circular portion of the curve gets sharper and the increased versine which is compensating the increased transition length is automatically obtained. The procedure is explained in example 5.2.

5.8 Local adjustment of curved alignment: When only a short stretch of the curve, say 100 to 150 meters long, gets disturbed it is neither necessary nor desirable to realign the entire curve. It would suffice if the versines over the disturbed portion only are adjusted. As per IRPWM also (see para 5.4), when the number of stations having station to station versine variation is less than 20% of the total number of stations on the curve, only local adjustments are to be carried out. If such an attention is not given in time, it will lead to disturbance of versines on the nearby stretches also in due course of time, necessitating realignment of the entire curve.

Further, if the running of a curve is bad and the resources for the complete realignment are not available, only part of the curve may be attended. To get the maximum benefit from the limited efforts put in a curve, tackling worst few locations as local adjustment provides immediate relief.

- Procedure for Local Adjustment: The local adjustment can be done by solving a portion of the curve by keeping the basic principles of realignment enumerated above in mind. This is two stage procedure, similar to the realignment problem:
 - Consider only the existing versines of the disturbed portion of the curve. While taking the disturbed portion, one or two stations on either side of the disturbed stretch where the versines are not disturbed much should be included.
 - Find out the total of the existing versines for the disturbed curve.
 - Divide the sum of existing disturbed versines by the number of stations on the disturbed curve and the same is the proposed versine at all the stations of the curve.
 - Calculate the first summation and second summation of the versine difference as already described for the disturbed curve only.

• If there is any residual slew at the last station, eliminate the same by application of a suitable correcting couple as already explained.

The procedure is explained in example 5.3.

- **5.9 Attention to transitions:** Due to the lateral curving and centrifugal/ centripetal forces, curves require more maintenance as compared to straight track. Within the curve, transition portion requires maximum attention due to following reasons:
 - There is a continuous change in the curvature/ versines/ cross levels and the lateral forces are varying constantly in the transitions.
 - There is jerk at the entry and exit of the curves due to sudden introduction of the versines.

• Parameters of versine and change in cross levels have maximum permissible values from comfort/ safety considerations. These values are same for the circular portion and the transition portion. In the transition portion of curves, there is already a designed change in curvature and cant. This means that the margin for deterioration in the parameters on the transitions is correspondingly reduced and the transition portions are required to be attended more frequently as compared to the other portions of track.

To attend to the transition portion of a curve alone, we shall take the end portion of curved track which is approximately 1.5 times transition length, starting from the apparent tangent point. Further procedure is similar to the local adjustment problem described in para 5.8 above, except that the proposed versines shall vary uniformly in the transition portion and shall be uniform in the circular portion. The typical curve in plan where the transitions are disturbed shall look like:



Figure 5.15: Disturbed transitions in a curve

Since the length near transitions is disturbed, we take 1.5 times length of curve for realignment, x equal to O-X. The point O is the apparent tangent point (i.e. the point where the versines are almost zero). The curve upto point X can have some portion of circular curve also.

The proposed versines for the disturbed curve O-X shall be designed such that the slew at X is zero. This will ensure that the length of curve to be tackled is minimized and the transition length gets attended. Therefore, the SS of versine difference shall be zero at station X.

To simplify the computations, as earlier, let us consider an equivalent circular curve of length N having same sum of versines as the disturbed curve. Later on, we can add the length of transition. The transition curve length, L, is already known from the speed considerations.

Let us assume V is the versine for the equivalent circular curve.

Since we are considering the curve upto station X only where the last station is having non zero versine value, FS of the proposed curve upto station X shall be equal to the F.S. of Ve up to (X-1)

stations + $\left(\frac{V}{2}\right)$

Since the sum of versines for the existing and proposed curves shall be same, length of equivalent circular curve will be given by $x^*V = FS$ of v_e upto $stations(X-1) + \frac{V}{2}$

i.e. Length,
$$x = \frac{FS \ of \ v_e}{V}$$
 up to stations $(X-1) + \frac{V}{2}$

By equating offset at X for disturbed and proposed curves, we get:

$$O_x = 2SS$$
 of v_e up to $X = Vx^2 + \frac{V(L^2 - 4)}{2}$

$$O_x = V \left[\frac{FS \ of \ V_e \ FS \ of \ V_e \ up to \ (X-1) + \ \frac{V}{2}}{V} \right]^2 + \frac{V(L^2 - 4)}{2}$$

This becomes a quadratic equation for V and can be solved. The beginning of curve can be found out as:

$$X - N - \frac{L}{2} = X - \frac{F.S. \ of \ V_e \ up to \ (X - 1) + \ \frac{V}{2}}{V} - \frac{L}{2}$$

Versines on transitions can now be designed as done earlier with minor adjustment for total versines up to stations (X-1), if necessary.

The slews now can be worked out in the usual manner.

The procedure will be clearer from example 5.4.

The transitions require attention at closer intervals compared to the other parts of the track and this method can help the work to be done mathematically rather than by eyesight, which can lead to further disturbance of the curve. The regular attention to the transitions can help reduce the need for complete realignment at later date.

- **5.10** Use of Computer Programs for Realignment of Curves: The realignment process described in para 5.6 above is convenient to be done by hand only when the number of stations is less, say up to 40. Further, the hand calculations suffer from following limitations:
- Manual calculations are error prone

- The procedure is tedious and there is reluctance on part of engineer to do the calculations again and again. Whenever we are carrying out the curve realignment, it is not always possible that we have the best solution in the first attempt. Therefore, if we want to have a few alternatives and wish to compare these, the calculations have to be done again and again.
- Considerations of restrictions such as obligatory points or restriction on maximum slews makes the problem even more complex and very difficult to be solved by hand.
- As the number of stations increase, the calculations get very tedious.

To remedy the situation, the computers can and must be used. Computers provide freedom from donkey-labour part of the solution and the engineer can concentrate on the quality of output and refining the same. Using computers, solution to curve realignment problem from starting is obtained in a matter of minutes and iterations can be done in a matter of seconds. Engineer can easily see various alternatives and compare them to choose the best suited solution.

The modern day computer programs have user friendly interface and their operation is quite easy with visual guides and help available for the same. In nut shell, computer programs make the realignment problem lot easier to tackle.

However, a note of caution. Use of computer programs does not eliminate the need for a permanent way engineer. The realignment solution cannot be done by just anybody. A keen sense of geometry, knowledge of the permanent way and the various site conditions/ restrictions is required to appreciate the various solutions that are obtained from the computer programs, and to choose the most convenient solution.

As already mentioned, computer programs mostly use the same procedures as enumerated above in para 5.6. The knowledge of this theory is not the pre-requisite for operating the computer programs but if the permanent way engineer is having the knowledge of the basics of how the computer program is working, he/she can better appreciate the results given by the computer programs and also, which parameter to alter so as to control the solution and get a more convenient and acceptable solution.

- 5.10.1 **How Computer Programs work:** As mentioned above, the computer programs basically use the same algorithm or basic logic as the detailed procedure given in the para 5.6 above while meeting all constraints or obligatory points on the curve. Further, the computer programs do the optimization of the solution. The optimization of the realignment solution is generally done to:
 - (i) Reduce the maximum slew so that the realignment solution can be practically implemented.
 - (ii) Reduce the total slewing effort

A variety of computer programs have been developed by the Indian Railway engineers and are in use by the field engineers. The most popular amongst the available programs are:

- (i) RC 100 or RECUR 100 developed by Mr. V.K. Vaish
- (ii) Program by Dr. M. Sheshagiri Rao, Retd Chairman, RITES (IRSE Officer of 1957 exam batch)
- (iii) Program by Shri M.S. Ekbote, Retd. AMCE(Works) Rly.Bd. (IRSE Officer of 1965 exam batch)
- (iv) Program in excel sheet by Shri Venkateswara Rao, (IRSE Officer of 1988 exam batch)
- (v) Some tamping machines such as 3X machine and new DUOMATIC machines are having on-board computers and employ a software called WINALC. These machines take a measuring run and give solution for realignment after the same.

(vi) Other computer programs are available from the dedicated railway layout software such as the MXRAIL.

The different programs have different algorithms and the parameters used for optimization are different:

- (i) **Maximum value of slew (positive) shall be nearly equal to maximum value of slew (negative).** This optimization is used by the program by Shri M.S. Ekbote.
- (ii) **Absolute sum of all slews shall be minimum:** This optimization is used by RC100.
- (iii) **Root mean square value of slews shall be minimum:** This optimization is used by the program by Dr. M. Sheshagiri Rao.
- (iv) Limit the maximum slewing effort at any one station: This optimization is used by the program developed in excel developed by Shri K. Venkateswara Rao. In his own words, the principle is "propose the versines which are same as existing versines at all stations except at two stations which cause the maximum versine variation and do usual double summation calculations. Repeat the process a number of times". This algorithm has been incorporated in one of the options in the realignment problem developed by Sh M S Ekbote.
- (v) Averaging method: This is not a holistic approach to solution of the curve but in some cases where the existing curve is badly distorted and there are severe site restrictions, this method will give some solution which is practical. In this method, even though the resulting geometry is not very good, it is much better than the smoothening mode tamping that is the only option left otherwise. In this method proposed versines are taken as average versine of 3 stations and successive iterations are done at user's choice. This algorithm has been incorporated in one of the options in the realignment problem developed by Sh M S Ekbote.
- (vi) Divide the curve into segments: The compounding method in the realignment program developed by Sh M S Ekbote gives an option wherein the curve can be divided into segments, and the realignment solution is found out by converting the curve into a compound curve. The segments have to be chosen such that the curve is separated at points where the average versines of the curve change. Some amount of engineering judgment is involved in choosing the compounding points. With some trial and error, the optimum location for the compounding points can be found out and an acceptable solution can be obtained. This algorithm has been incorporated in one of the options in the realignment problem developed by Sh M S Ekbote.

All the above optimization methods give different benefits to the permanent way engineer. The first method of optimization balances the maximum value of outward slew with the maximum value of inward slew. Since the absolute value of slew is very important constraint in open line working, the method gives good solutions.

The second method of optimization reduces the total slewing effort or the effort required to slew the curve, even though a few points may require higher slews. This method controls the sum total of slews.

On the other hand, the third method of optimization reduces the total slewing effort by controlling the root mean square of slews.

The fourth and fifth methods are sometimes the only feasible solutions in open line electrified territory, even though in these methods, the curve cannot be improved beyond a certain point.

The sixth method is a very powerful tool and the permanent way engineer carrying out the realignment can very accurately control the solution of the curve. Depending on the type of curve, and the extent of disturbance, any of the approaches can give better or acceptable solution to the permanent way engineer. 5.10.2 Which computer program to use?: As discussed above, at least four different programs (and more sub-program options) are available, which have been extensively used by Indian Railways people over the years. Based on the discussions held with the authors of the programs and guest officers attending the various training programs at IRICEN, as also the field engineers who regularly use the computer programs for realignment, it has been concluded that the realignment of curve has many different solutions and the engineer has to decide whether a solution given by a program is practical or not. The engineer has to work hard and understand the curve and its geometry properly and vary his specifications, constraints and boundary conditions to get a few solutions before he gets an alternative which is easy to be implemented. Here, it is brought out that these constraints and boundary conditions are for the open line, engineers who work under the constraints of having to work only in limited traffic blocks and where OHE and other constraints are guiding factors. In the case of new constructions and gauge conversion works, the curves shall be laid to perfect geometry as designed and the realignment solution aiming at true circular curve shall only be followed, even if the same means extra efforts to improve badly laid/adjusted curve during construction phase. If the curves are laid at the initial stage itself with compounding or averaging etc, these will give lot of trouble to the maintenance engineers and rectification of geometry will become very difficult once the trains start moving on the track.

The program for realignment of curves written by Shri M.S. Ekbote has various alternatives for realignment and gives good solutions for construction as well as open line. This program is available in the members' area in IRICEN website <u>http://www.iricen.gov.in</u>. The latest version of the program is dated 25-01-2009. Detailed instructions for use of this program and basic logic are given at Annexure-I.

- **5.11** Attention to Curves in Electrified Sections: In electrified section, there is a limitation posed by the presence of the OHE masts and the track cannot normally be slewed in any direction more than 75-100 mm, or even less. Any realignment work in electrified section shall commence only after the electrical department has been informed in advance and it has confirmed the feasibility of the same. For larger slews, the electrical department has to adjust the OHE and/ or shift OHE masts. To tackle this problem, the realignment solution has to be found out where the slews are within limits at all stations. For longer curves or for very bad curves, it might not always be possible to adopt the realignment solutions worked out manually or through computer programs in electrified section due to the above restrictions. such situations shall be tackled as follows:
 - i. **Partial improvement by limiting the maximum slews** using computer programs available (The solution by averaging method or iterations method in the Realignment program by Sh M S Ekbote or the method by iterations developed by Sh Venkateswara Rao).
 - ii. **Improvement by compounding the curve** can also be tried. The curve may be divided into a number of small segments and the solution for each segment can be obtained which is practical and meets the various site/ operational constraints. The segments will have uniform curvature and the curvature shall be in close proximity to the curvature of the stretches on either side. The solution by compounding in the realignment program developed by Mr M S Ekbote gives such a solution which can be used. This solution gives very good results in some curves which are disturbed badly in some part only and is OK in other parts.
 - iii. **Part Curve Solution:** The attending to part of curve is sometimes very beneficial as the complete curve is not bad and un-necessarily the complete curve shall not be attended due to this problem. For this, earlier the effort was required to be made manually only. However, for the first time, the program developed by Mr M S Ekbote gives an option for tackling only part of

the curve. This option works well if the part is not very bad and gives error if the portion selected for improvement is not feasible to be corrected. In such a case, the length of curve to be tackled has to be increased.

5.12 Getting better solutions using the software by Sh M S Ekbote:

The solution given by the computer program developed by Sh M S Ekbote can be improved to get reduced solutions by using the following methods:

- The minimum transition length is fixed by the speed and other considerations. However, we can always provide longer transitions. In the computer program, there is a facility of having unequal transition lengths on the either end of the curve. By changing the transition length, we can have some control over the slews and we can do iterations and choose from amongst the various options.
- Sometimes, there are locations where slews are excessive, and even after making all trials, the slews are not coming within acceptable limits. Such a solution is not acceptable and feasible. However, we can manipulate the powers of computer program to give us an acceptable solution by inserting an imaginary obligatory point at the location and limit the slew to acceptable limit. By doing this, the curve gets distorted, but the result is feasible at site and sometimes may be the best option.

5.13 Examples of Curve Realignment

5.13.1 Example 5.1: The versines have been surveyed for a curve as given in table below. Find out the realignment solution by string lining method.

Stn	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Versine	0	3	8	7	9	15	6	2	5	-4	15	16	18	20	8	7	3	0

Solution:

STEP I: Compute first and second summations for existing versines: Prepare summation Table – for existing versines, Ve as below. (It is desirable to make the table as below for error free calculations).

Stn.	Ve	F.S. of	S.S. of
No.		ve	ve
0	0	1	
1	3	V	>0
		3	
2	8		$>_3$
		11	
3	7		14
		18	
4	9		32
		27	
5	15		59
		42	

Stn.	Ve	F.S. of	S.S. of
No.		Ve	Ve
6	6		101
		48	
7	2		149
		50	
8	5		199
		55	
9	-4		254
		51	
10	15		305
		66	
11	16		371
		82	
12	18		453
		100	
13	20		553
		120	
14	8		673
		128	
15	7		801
		135	
16	3		936
		138	
17	0		1074

STEP II: Compute Beginning of curve, length of curve and proposed versines: In the same summation table of step I, do further calculations for the beginning of curve, length of curve and proposed versines as shown below. The transition length for the curve is taken as 4 station units.

Stn. No.	Ve	F.S. of Ve	S.S. of Ve	FURTHER CALCULATIONS
0	0 \	7		STEP a : Calculate station/chainage of CC using eq (5.1)
		~	~	$= n - \frac{SSV_e upto n}{n}$
1	3		0	FSV_e upto n
		3		
2	8		3	$\therefore 17 - \frac{1}{138} = 17 - 7.8 = 9.2$

Stn.	Ve	F.S.	S.S.	FURTHER CALCULATIONS						
No.		Ve	OI VE							
		11		STEP b: By I	STEP b : By Interpolation in the table, Offset at center of curve, O_c at					
3	7		14	station 9.2	station 9.2					
		18		= 2 (254+0.2	x51) = 528					
4	9		32	STEP c: Usir	STEP c: Using eq (5.7), \therefore 528 = $\left(\frac{138 \times 138}{2}\right) \times V + \frac{V(L^2 - 4)}{2}$					
		27			$(4V^2)$ 12					
5	15		59	Taking L = 4	we get					
		42		$528 = \frac{69 \times 6}{2}$	$\frac{59}{-} + V$					
6	6		101	V						
		48		Solving for V	we get					
7	2		149	V _p (i.e. propos	sed versine) = 9.2					
		50		STEP d: Usir	STEP d: Using eq (5.6), The length of equivalent circular curve					
8	5		199	$N = \frac{138}{15} = 15$ stations						
		55		9.2						
9	-4		254	Note: n is the station number of the last station on the disturbed curve.						
		51		\therefore The total length of transitioned curve= 15 + 4 = 19 stations						
10	15		305	\therefore Beginning of Curve = 9.2 – 19/2 = station no -0.3						
		66		and End of Curve = $9.2 + 19/2 =$ station no 18.7						
11	16		371	Slope of versine diagram in the transition portion = $9.2/4 = 2.3$						
		82		STEP e: Calo	culations of proposed versine	s shall be done as follows:				
12	18		453	Station	Calculated versine	Rounded off to				
		100		-1		0.0				
13	20		553	0	0.3 x 2.3 = 0.69	0.7				
		120		1	0.69 + 2.3 = 2.99	3.0				
14	8		673	2	2.99 + 2.3 = 5.29	5.3				
		128		3	5.29 + 2.3 = 7.59	7.6				
15	7		801	19		0.0				
		135		18	0.7 x 2.3 = 1.61	1.6				
16	3	<u> </u>	936	17	1.61 + 2.3 + 3.91	3.9				
		138		16	3.91 + 2.3 + 6.21	6.2				
17	0		1074	15	6.21 + 2.3 = 8.51	8.5				
					Total versine over transitions:	36.8				

Having known $V_{\rm p},\,$ EC, BC and versine slope, the versines on transition can be conveniently worked out by drawing a versine trapezium as shown below.





STEP III: Calculate first and second summation of versine differences: Proposed versines shall be filled in table opposite existing versines. Computation of fist summation, second summation of the versine difference shall be done as shown in step I. The final results are shown below:

SN	Ve	Vp	Vp-Ve	F.S. of	S.S. of	
				(Vp-Ve)	(Vp-Ve)	
-1	0	0	0			
0	0	0.7	0.7	0	0	
1	3	3	0	0.7	+0.7	
2	8	5.3	-2.7	0.7	+1.4	
3	7	7.6	0.6	-2.0	-0.6	
4	9	9.2	0.2	-1.4	-2	
5	15	9.2	-5.8	-1.2	-3.2	
6	6	9.2	3.2	-7.0	-10.2	
7	2	9.2	7.2	-3.8	-14	
8	5	9.2	4.2	+3.4	-10.6	
9	-4	9.2	13.2	+7.6	-3	
10	15	9.2	-5.8	+20.8	+17.8	
11	16	9.2	-6.8	+15.0	+32.8	
12	18	9.2	-8.8	+8.2	+41	
13	20	9.2	-10.8	-0.6	+40.4	
14	8	9.2	1.2	-11.4	+29	

15	7	8.5	1.5	-10.2	+18.8
16	3	6.2	3.2	-8.7	+10.1
17	0	3.9	3.9	-5.5	+4.6
18	0	1.6	1.6	-1.6	+3
19	0	0	0	0.0	+3
Sum	138	138	0.0		

STEP IV: Application of correcting couple: There is a residual second summation of +3. This very small value can be neglected but as it is of no significance in field. However, for the sake of accuracy and explanation in this example, let us eliminate the same by using correcting couples.

Applying rules for applying correcting couple described in para 5.6.3.6 above, we require to put in correction with negative value in the initial stations and one with positive value in latter stations. Since correcting couple shall be as small as possible, as far away as possible and equal and opposite, let us put a small correcting couple of -0.1 mm at station no 0 and corresponding +0.1 mm at station no 18. (farthest stations in the curve)

By mental arithmetic, we can see that the second summation of this correcting couple at last station will be -1.8 mm (This can be calculated by value of the correction* distance between the two corrections, i.e. -0.1 mm * 18).

This is less than 3.0 mm, so we require another correcting couple, with second summation 3.0 mm - 1.8 mm=1.2 mm. Let this be also of value 0.1 mm, and provided between stations 3 and 15 (The second summation of this correcting couple is -0.1 mm * 12=-1.2 mm)

Proceeding further from the computations shown in step III, the computations after the correcting couples are applied are as below:

SN	Ve	Vp	S.S. of (Vp- Ve)	Vc	F.S. of Vc	S.S. of Vc	Half slews = S.S. of (Vp-Ve) + S.S. of Vc	Slews: 2 * [SS of (Vp – Ve) + SS of Vc]	Round- ed off slews (mm)	Final Vp
-1	0	0	-			-	0	0	0	
0	0	0.7	0	-0.1	0.0	0	0	0	0	0.6
1	3	3	+0.7		-0.1	-0.1	0.6	1.2	1	3.0
2	8	5.3	+1.4		-0.1	-0.2	1.2	2.4	2	5.2
3	7	7.6	-0.6	-0.1	-0.1	-0.3	-0.9	-1.8	-2	7.5
4	9	9.2	-2		-0.2	-0.5	-2.5	-5.0	-5	9.2
5	15	9.2	-3.2		-0.2	-0.7	-3.9	-7.8	-8	9.2
6	6	9.2	-10.2		-0.2	-0.9	-11.1	-22.2	-22	9.2
7	2	9.2	-14		-0.2	-1.1	-15.1	-30.2	-30	9.2
8	5	9.2	-10.6		-0.2	-1.3	-11.9	-23.8	-24	9.2
9	-4	9.2	-3		-0.2	-1.5	-4.5	-9.0	-9	9.2

10	15	9.2	+17.8		-0.2	-1.7	+16.1	+32.2	+32	9.2
11	16	9.2	+32.8		-0.2	-1.9	+30.9	+61.8	+62	9.2
12	18	9.2	+41		-0.2	-2.1	+38.9	+77.8	+78	9.2
13	20	9.2	+40.4		-0.2	-2.3	+38.1	+76.2	+76	9.2
14	8	9.2	+29		-0.2	-2.5	+26.5	+53.0	+53	9.2
15	7	8.5	+18.8	+0.1	-0.1	-2.7	+16.1	+32.2	+32	8.6
16	3	6.2	+10.1		-0.1	-2.8	+7.3	+14.6	+15	6.2
17	0	3.9	+4.6		-0.1	-2.9	+1.7	+3.4	+3	3.9
18	0	1.6	+3	+0.1	0.0	-3.0	0	0	0	1.7
19	0	0	+3		0.0	-3.0	0	0	0	0

Note:

- 1. The slew at Stn. 9 is 9 mm and at 10 is +32mm. Therefore, the slew at center of curve, i.e. at station 9.2, the slew is almost zero as designed.
- 2. The maximum slew is only +78 mm which is almost within practical limits of working of tamping machines.

5.13.2 Example 5.2: If the station 12 in the curve data given in example 5.1 happens to be obligatory and no slew is permitted, how will the solution change?

Solution:

The station 12 had the maximum slew of +78 mm as seen in the solution above. If no slew is permitted at this location, the procedure for finding out the solution will be as follows:

STEP I: Equate the offset of existing curve with that of proposed curve at the station no 12: The example 5.1 has been done by equating the offset from the tangent at the center of curve for the existing curve and the proposed curve. Here, since there is restriction of slew at station no 12, existing offset at Station 12 shall be equated to the offset at station 12 due to proposed curve. As discussed earlier, the offset for the proposed curve shall be in two parts, offset for equivalent circular curve and the shift due to proposed transition.

It may be noted that now the designed versine will be altered in such a way that the proposed transitioned curve passes through station 12 of existing curve and due to the same, the slews will increase. Therefore, the objective of passing the proposed curve through a specific point of the existing curve can be achieved at the cost of increased maximum slew.

As in the example 5.1, Offset of the existing curve at Station 12, i.e. $O_{12} = 2^*$ second summation of the existing versines upto station no $12 = 2 \times 453 = 906$.

Now, Designed offset at station 12 due to proposed circular curve = VT^2 where T is the tangent length upto station no 12, which is taken equal to the length of circular curve due to the large radius of the railway curves.

Now, $T = \frac{N}{2} + (12 - CC)$ where $N = \frac{Total \ F.S. \ of \ V_c}{V}$

Designed offset due to $shift = (V) \quad \frac{(L^2 - 4)}{12} = V \quad sin \, ce \quad L = 4 \quad as \quad earlier$ $\therefore \quad 906 \quad = \quad \left(\frac{138}{2V} + (12 - 9.2)\right)^2 \times V \quad + \quad V$

Solving this, we get V = 11.08

$$\therefore$$
 N = $\frac{138}{11.08} = 12.46$ and N'= N + L = 16.46

And, Versine slope = $\frac{11.08}{4} = 2.77$

Using equation (5...), Beginning of curve, $BC = 9.2 - \frac{16.46}{2} = 0.97$

Using equation (5...), End of curve, $EC = 9.2 + \frac{16.46}{2} = 17.43$

Versine diagram and design of versines on transition for the proposed curve.

Table showing calculations of proposed versines.

Station	Calculated versine	Rounded off to
0		0.0
1	$=\frac{.03\times11.08}{4}=.08$	0.1
2	= 0.8 + 2.77 = 2.85	2.8
3	= 2.85 + 2.77 = 5.62	5.6
4	= 5.62 + 2.77 = 8.39	8.4
18	0.0	
17	$=\frac{0.43\times11.08}{4}=1.19$	1.2
16	= 1.19 + 2.77 = 3.96	4.0
15	= 3.96 + 2.77 = 6.73	6.7
14	= 6.73 + 2.77 = 9.5	9.5
	Total:	38.3

Table:

Taking V_p as 11.1 instead of 11.08 the total versines on circular position will be 99.9, so to adjust the total, versine at stations 5 and 13 will betaken as 11 (this being necessary as the calculations are limited to first decimal place) the result can be seen in the following table No...

SN	Ve	Vp	S.S. of Vp-Ve	Vc	S.S. of Vc	Final Vp	Final slews rounded off to nearest mm
0	0	0	0			0	
1	3	0.1	0			0.1	
2	8	2.8	-2.9			2.8	-6
3	7	5.6	-11			5.6	-22
4	9	8.4	-20.5			8.4	-41
5	15	11	-30.6			11.0	-61
6	6	11.1	-44.7	-0.1		11.0	-89
7	2	11.1	-53.7		-0.1	11.1	-107
8	5	11.1	-53.6		-0.2	11.1	-107
9	-4	11.1	-47.4		-0.3	11.1	-95
10	15	11.1	-26.1		-0.4	11.1	-53
11	16	11.1	-8.7		-0.5	11.1	-18
12	18	11.1	+3.8		-0.6	11.1	+6
13	20	11	+9.4	+0.1	-0.7	11.1	+17
14	8	9.5	+6.0		-0.7	9.5	+11
15	17	6.7	+4.1		-0.7	6.7	+7
16	3	4	+1.9		-0.7	4	+2
17	0	1.2	+0.7		-0.7	1.2	0
18	0	0	+0.7			0	0

Table showing results with an obligatory point at station 12

Table –

Note:

- 1. Slew at station 12 is only + 6mm as against 0.
- 2. The max. slew is 107 mm at stations 7 & 8
- 3. The designed versine is + 11.1 as against 9.2 earlier. This means that the curve has got sharper than earlier due to the obligatory point restriction.
- 4. The error of + 6 in SS of (Vp-Vc) can be safely ignored, but it is eliminated for the sake of accuracy, by application of a correcting couple without disturbing the versines very much.

5.13.3 Example 5.3: Versines recorded in portion of a curve is shown in table below. What action is required to be taken? Find out the solution to the problematic stretch.

Stn	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Ve	30	31	32	29	30	26	30	28	40	32	40	30	14	12	28
Stn	34	35	36	37	38	39	40	41	42						
Ve	40	46	26	28	30	32	29	28	30						

Solution: On scrutiny of the existing versines, we see that the complete curve is not bad. Only a portion of the curve is having station to station versine difference. This is a problem which shall be tackled by local adjustments and not complete ralignment. The variation is seen between stations 24 to 36. Here, the choice of the stations between which the curve is considered disturbed is by engineering judgment and we can be consider the curve to be disturbed from station no 23 or 25 without affecting the result of curve much. However, longer the curve chosen for adjustment more will be the length of track to be attended. As per principles of realignment, we take a few stations on either side of affected portion and find out a solution for local adjustment of curve at stations 23 to 38 as below.

STEP I: The sum total of the versines in station 23 to 38 is 480. There are total 16 stations, so the proposed versine shall be 30. Using the proposed versines, let us find out the versine difference, first summation and second summations for stations 23 to 38.

STEP II: The second summation of the versine difference at the station no 38 comes to +12, so correction of -1 mm is applied at station no 25 and corresponding correction of +1 mm at station no 37 (so that the second summation of the correcting couple is -1 mm * 12 = -12 mm, as required). Second summation of the correcting couple is found out.

STEP III: The second summation of versine difference in Step I and the second summation of correcting couple in Step II shall be added up to get the final second summation. Twice this value gives the slews at each of the stations.

Stn	Ve	Vp	VP-	F.S.	S.S.	Vc	F.S.	S.S.	Final	Full	Final
No			ve	Vp- Ve	Vp- Ve		VC	vc	Slew	Siew	versine
19	30								0	0	30
20	31								0	0	31
21	32								0	0	32
22	29								0	0	29
23	30	30	0						0	0	30
				0							
24	26	30	+4		0				0	0	30
				+4							
25	30	30	0		+4	-1			+4	+8	29
				+4			-1				
26	28	30	+2		+8	0		-1	+7	+14	30
				+6			-1				
27	40	30	-10		+14	0		-2	+12	+24	30
				-4			-1				
28	32	30	-2		+10	0		-3	+7	+14	30
				-6			-1				
29	40	30	-10		+4	0		-4	0	0	30

Computations for local adjustment of curve as per above procedure are done as follows:

				-16			-1				
30	30	30	0		-12	0		-5	-17	-34	30
				-16			-1				
31	14	30	+16		-28	0		-6	-34	-68	30
				0			-1				
32	12	30	+18		-28	0		-7	-35	-70	30
				+18			-1				
33	28	30	+2		-10	0		-8	-18	-36	30
				+20			-1				
34	40	30	-10		+10	0		-9	+1	+2	30
				+10			-1				
35	46	30	-16		+20	0		-10	+10	+20	30
				-6			-1				
36	26	30	+4		+14	0		-11	+3	+6	30
				-2			-1				
37	28	30	+2		+12	+1		-12	0	0	31
				0			0				
38	30	30	0						0	0	30
39	32								0	0	32
40	29								0	0	29
41	28								0	0	28
42	30			1					0	0	30
				1					1		

5.13.4 Example 5.4:

While solving the example 5.1, the transition length was assumed to be 4. If the speed on the section is proposed to be increased, the bottle neck is the transition length and the same is to be increased to 6 station units. Find out the slews required for this situation.

Now, if we provide the longer transition, the shift will also increase. The shift increases by square of transition length and the same will increase by 2.25 times in this case where the length of transition is 1.5 times the original length. The entire curve will have to be shifted inwards if we follow this. However, we know that the curve shifts outwards If the radius is reduced, so a method of reducing the amount of slews is that the circular curve be made sharper (radius may be reduced). Therefore, for minimum slews, the existing offset at CC is equated to offset at CC due to proposed circular curve plus the offset due to the revised shift.

From example 5.1, we already know that for the existing curve, CC = 9.2 and

Offset at center of curve, O_c at 9.2 = 528 as before

Equating this to proposed shift at CC, we get

$$528 = \frac{69 \times 69}{V} + \frac{V(36 - 4)}{12}$$
$$= \frac{69 \times 69}{V} + \frac{8}{3}V$$

Solving this quadratic equation for V, we get V = 9.76, say 9.8. (Since the versine in circular portion, 9.8 is more than 9.2 in example 5.1, it is clear that the radius has been reduced)

Length of equivalent circular curve i.e.

$$N = \frac{138}{9.76} = 14.14(N = (N + L) = 14.14 + 6 = 20.14)$$
$$BC = 9.2 - \frac{20.14}{2} = -.87$$
$$EC = 9.2 + \frac{20.14}{2} = 19.27$$

Versine diagram and design of versines for the proposed curve are as follows:.



Versines on transitions i.e. at stations 0 to 5 and 14 to 19 can be worked out in a manner similar to the one shown in example 5.1. The versines at station nos -1 and 20 are zero. The results of computed slews are shown below:

S No	Ve	Vp	S. S. of Vp-Ve	Vc	S. S. of Vc	Final Vp	Final slews rounded off to nearest mm
-0	0	0	0				
0	0	1.4	0	-0.1		1.3	
1	3	3	+1.4		-0.1	3	+3

Table showing the results for increased transition length.

S No	Ve	Vp	S. S. of Vp-Ve	Vc	S. S. of Vc	Final Vp	Final slews rounded off to nearest mm
2	8	4.7	+2.8	-0.1	-0.2	4.6	+5
3	7	6.3	+0.9	-0.1	-0.4	6.2	+1
4	9	7.9	-1.7		-0.7	7.9	-5
5	15	9.5	-5.4	+0.1	-0.1	9.6	-13
6	6	9.8	-14.6		-1.2	9.8	-32
7	2	9.8	-20.0		-1.4	9.8	-43
8	5	9.8	-17.6		-1.6	9.8	-38
9	-4	9.8	-10.4		-1.8	9.8	-25
10	15	9.8	+10.6		-2.0	9.8	+17
11	16	9.8	+26.4		-2.2	9.8	+48
12	18	9.8	+36		-2.4	8.8	+67
13	20	9.7	+37.4	+0.1	-2.6	9.8	+70
14	8	8.6	+28.5		-2.7	8.6	+5.1
15	7	6.9	+20.2		-2.8	6.9	+35
16	3	5.3	+11.8	+0.1	-2.9	5.4	+18
17	0	3.7	+5.7		-2.9	3.7	+6
18	0	2	+3.3		-2.9	2	+1
19	0	0.4	+2.9		-2.9	0.4	0
20	0	0					

Note: 1) even though the transition length is increased by 1.5 times, the maximum slew is only + 70. The slew at station 9.2 is almost zero, the slew at Station 9 is -25 and at Station 10 is + 17

2) The SS of versine difference of +2.9 at last station is eliminated by application of small correcting couple, for the sake of accuracy of calculations. (Otherwise this can be left as it is without affecting the results in any manner)

3) The versine in the central portion of the curve in example 5.1 was 9.2, which has now changed to 9.8, indicating increase in degree or sharpness of curve.

5.13.5	Example 5.5:	A curve has	the following	readings	at one end:
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Stn No	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ve	0	7	6	5	15	12	24	14	25	38	43	26	25	32	31	29	30	31

The versines are to be corrected .

Stn. No.	Ve	F.S. of Ve	S.S. of Ve	
-2	0_			Transition length = 80m i.e. (8 stations)
		0		Portion to be tackled from station
-1	7		\geq_0	N02 to station No.11
		7	<u> </u>	Offset at station no 11, O ₁₁ = 2 x 1166 = 2332
0	6		\Rightarrow_7	
		13		
1	5		20	
		18		$\begin{bmatrix} 240 & V \end{bmatrix}^2$
2	15		38	$2332 = V \left[\frac{240 + \frac{1}{2}}{2} \right] + \frac{V(L^2 - 4)}{2}$
		33		V 12
3	12		71	
		45		V ² -398.48V+10971.43=0
4	24		116	Solving for V, we get
		69		V=29.76 say $\frac{29.8}{2} = 3.7mm$
5	14		185	8
		83		$BC=11-\frac{240+14.9}{2}-4$
6	25		268	29.8
		108		= -1.554
7	38		376	Versine at Station – $1 = 0.554 \times 3.7$
		146		= 2.064
8	43		522	Say 2.1
		189		
9	26		711	
		215		
10	25		926	
		240		
11	32		1166	
12	31			
13	29			
14	30			
15	31			



Figure 5.18

Stn.	Ve	Vp	Vp-Ve	F.S.	S.S.	F.S.	Final	Full	Final
NO.				Vp-Ve	vp-ve	VC	slew	Siew	vp
-2	0	0	0						
				0					
-1	7	2.1	-4.9		0			0	2.1
				-4.9					
0	6	5.8	-0.2		-4.9			-10	5.8
				-5.1					
1	5	9.5	+4.5		-10			-20	9.5
				-0.6					
2	15	13.2	-1.8		-10.6			-21	13.2
			-2.4						
3	12	16.9	+4.9		-13			-26	16.9
				+2.5					
4	24	20.6	-3.4		-10.5			-21	20.6
				-0.9					
5	14	24.3	+10.3		-11.4			23	24.3
	1			+9.4	1				
6	25	28.0	+3		-2			4	28
	1			+12.4	1				

Based on above computations, the solution can be worked out as follows:

Stn. No.	Ve	Vp	Vp-Ve	F.S. Vp-Ve	S.S. Vp-Ve	F.S. Vc	Final ¹ /2 slew	Full slew	Final Vp
7	38	29.8	-8.1		+10.4			+21	29.9
		+.1		+4.3					
8	43	29.8	-13.1		+14.7			+29	29.9
		+.1		-8.8					
9	26	29.8	+3.9		+5.9			+12	29.9
		+.1		-4.9					
10	25	29.8	+4.9		+1			+2	29.9
		+1		0					
11	32	32			+0			+1	32
12	31	31							31
13	29	29							29
14	30	30							30
15	31	31							31

Note: a) 0.1 is added to versine at stations 7 to 10 to adjust the total to 240.

b) Residual slew of +2 mm at station 10 is neglected and is adjusted locally by giving a slew of + 1 at station 11.

c) The other end can also be tackled in similar manner.

-0-0-0-